OPTIMIZATION OF A THERMODIFFUSION APPARATUS WITH TRANSVERSE FLOWS

G. D. Rabinovich and A. V. Suvorov UDC 621.039.3

Two variants are examined for optimizing a system with transverse flows and are compared for energy efficiency with an ideal cascade.

It was shown in [1] that, in a thermodiffusion apparatus operating with transverse flows (Fig. 1a and b), the running concentrations in the top channel, where the object component of the binary mixture is enriched, are determined from the following formulas in the case of direct flow and counterflow, respectively:

direct flow

$$c_e = c_0\left\{\exp(-\varphi_p\xi) + \left(1+\frac{\varkappa_i}{\varkappa_e}\right)\exp(by_e)\,[1-\exp(-\varphi_p\xi)]\times\right.$$
$$\left.\times\left[\exp(by_e)+\frac{\varkappa_i}{\varkappa_e}\right]^{-1}\right\} + \frac{a}{b}\,\frac{\varkappa_i}{\varkappa_e}\left[1-\exp(-\varphi_p\xi)\right][\exp(by_e)-1]\left[\exp(by_e)+\frac{\varkappa_i}{\varkappa_e}\right]^{-1}, \tag{1}$$

counter flow

$$c_e = c_0\left\{\frac{\varkappa_i}{\varkappa_e}[\exp(by_e)-1]\exp(\varphi_c\xi + \left[\exp(\varphi_c)-\frac{\varkappa_i}{\varkappa_e}\right]\exp(by_e)\right\}\times$$
$$\times\left[\exp(\varphi_c+by_e)-\frac{\varkappa_i}{\varkappa_e}\right]^{-1} + \frac{a}{b}\,\frac{\varkappa_i}{\varkappa_e}[\exp(by_e)-1]\,[\exp(\varphi_c\xi)-1]\left[\exp(\varphi_c+by_e)-\frac{\varkappa_i}{\varkappa_e}\right]^{-1}, \tag{2}$$

where a and b are coefficients in a linear approximation of the quadratic term of the transport equation

$$c(1-c)\approx a+bc, \tag{3}$$

and

$$\varphi_p = \frac{b}{\varkappa_i}\,\frac{\exp(by_e)+(\varkappa_i/\varkappa_e)}{\exp(by_e)-1};\ \varphi_c = \frac{b}{\varkappa_i}\,\frac{\exp(by_e)-(\varkappa_i/\varkappa_e)}{\exp(by_e)-1}.$$

Formulated below are the conditions ensuring a minimum of energy expenditure for operation with the above schemes. In a plane or nearly plane apparatus,* the heat flow rate on a section of length dx (Fig. 1a)

$$dQ = \frac{\lambda}{\delta}\,\Delta TLdx = \frac{\lambda}{\delta}\,\Delta TBLd\xi. \tag{3a}$$

The total heat flow rate in the apparatus

$$Q = \lambda\Delta TB\int_0^1 \frac{L}{\delta}d\xi = \lambda\Delta TB\int_{c_0}^{c_{eh}}\frac{L}{\delta}\,\frac{dc_e}{dc_e/d\xi}. \tag{4}$$

Introducing the notation

$$y_e^* = y_e\,\frac{\delta^4}{L},\ \varkappa_e^* = \varkappa_e\delta^3, \tag{5}$$

*By nearly plane, we mean a cylindrical apparatus in which the ratio of the inside diameter of the outer cylinder to the outside diameter of the inner cylinder is less than 1.1.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 6, pp. 937-946, June, 1982. Original article submitted April 8, 1981.

0022-0841/82/4206-0627$07.50

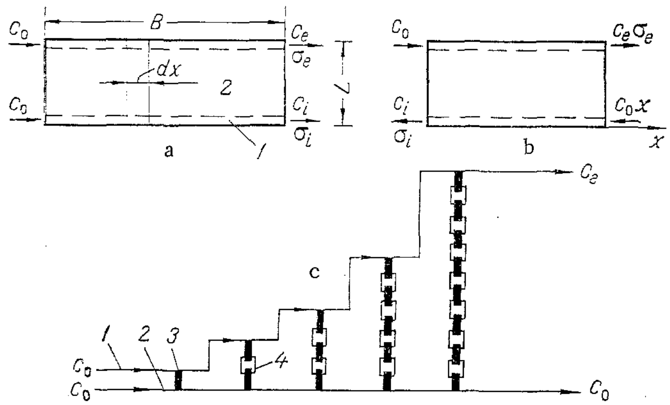

Fig. 1. Scheme of apparatus with transverse flows: a, b) direct flow and counterflow (1 — channels, 2 — separation part of apparatus); c) optimized scheme (1, 2 — supply of initial product, 3 — column, 4 — thermosiphons).

we obtain the following, instead of (4)

$$Q = \lambda \Delta T B \frac{\varkappa_e^*}{y_e^*} \int_{c_0}^{c_{ek}} \frac{y_e}{\varkappa_e} \frac{dc_e}{dc_e/d\xi}. \qquad (6)$$

We will replace the derivative $dc_e/d\xi$ in (6) using Eqs. (1) and (2):

in direct flow

$$\frac{dc_e}{d\xi} = c_0(1-c_0)\frac{1}{\varkappa_e}\exp(-\varphi_p\xi), \qquad (7)$$

in counter flow

$$\frac{dc_e}{d\xi} = \frac{c_0(1-c_0)}{\varkappa_e}\exp(\varphi_c\xi)\left[\exp(by_e) - \frac{\varkappa_i}{\varkappa_e}\right]\left[\exp(\varphi_c + by_e) - \frac{\varkappa_i}{\varkappa_e}\right]^{-1}, \qquad (8)$$

where we have made the substitution $a + bc_0 \approx c_0(1 - c_0)$, in accordance with (3).

The exponents $\exp(-\varphi_p\xi)$ and $\exp(\varphi_c\xi)$ can be expressed by means of (1), (2) through the running concentrations:

$$\exp(-\varphi_p\xi) = 1 - \frac{c_e - c_0}{c_0(1-c_0)}\frac{b\varkappa_e}{\varkappa_i}\left[\exp(by_e) + \frac{\varkappa_i}{\varkappa_e}\right][\exp(by_e) - 1]^{-1}, \qquad (9)$$

$$\exp(\varphi_c\xi) = 1 + \frac{c_e - c_0}{c_0(1-c_0)}\frac{b\varkappa_e}{\varkappa_i}\left[\exp(\varphi_c + by_e) - \frac{\varkappa_i}{\varkappa_e}\right][\exp(by_e) - 1]^{-1}. \qquad (10)$$

Substitution of (9) and (10) into (7) and (8) gives:

for direct flow

$$\frac{dc_e}{d\xi} = \frac{c_0(1-c_0)}{\varkappa_e} - (c_e - c_0)\frac{b}{\varkappa_i}\left[\exp(by_e) + \frac{\varkappa_i}{\varkappa_e}\right][\exp(by_e) - 1]^{-1}, \qquad (11)$$

for counter flow

$$\frac{dc_e}{d\xi} = \frac{c_0(1-c_0)}{\varkappa_e}\left[\exp(by_e) - \frac{\varkappa_i}{\varkappa_e}\right]\left[\exp(\varphi_c + by_e) - \frac{\varkappa_i}{\varkappa_e}\right]^{-1} + \frac{b}{\varkappa_i}(c_e - c_0)\left[\exp(by_e) - \frac{\varkappa_i}{\varkappa_e}\right][\exp(by_e) - 1]^{-1}.$$

Using (6), (11), and (12), we obtain:

$$Q_{\text{dire}} = \lambda \Delta T B \frac{\varkappa_e^*}{y_e^*}\int_{c_0}^{c_{ek}} y_e[\exp(by_e) - 1]\left\{c_0(1-c_0)[\exp(by_e) - 1] - \frac{b\varkappa_e}{\varkappa_i}(c_e - c_0)\left[\exp(by_e) + \frac{\varkappa_e}{\varkappa_i}\right]\right\}^{-1} dc_e, \qquad (13)$$

$$Q_{\text{coun}} = \lambda \Delta T B \frac{\varkappa_c^*}{y_e^*}\int_{c_0}^{c_{ek}} y_e\left[\exp(\varphi_c + by_e) - \frac{\varkappa_i}{\varkappa_e}\right][\exp(by_e) - 1]\times$$

$$\times\left\{\left[\exp(by_e)-\frac{\varkappa_i}{\varkappa_e}\right]\left\{c_0(1-c_0)[\exp(by_e)-1]+\frac{b\varkappa_e}{\varkappa_i}(c_e-c_0)\times\right.\right.$$
$$\left.\left.\times\left[\exp(\varphi_c+by_e)-\frac{\varkappa_i}{\varkappa_e}\right]\right\}\right\}^{-1}dc_e. \tag{14}$$

Finding the minimums of the functionals (13) and (14) is the condition ensuring a minimum of energy expenditures. We will henceforth examine the case when the rate of pumping through the lower channel is so great that the concentration in this channel may be considered everywhere constant and equal to c_0. This will occur when $\varkappa_i \to \infty$. Here, the difference between the direct flow and counterflow disappears and, instead of (13), (14), we will have

$$Q=\lambda\Delta TB\frac{\varkappa_e^*}{y_e^*}\int\limits_{c_0}^{c_{ek}}\frac{y_e[\exp(by_e)-1]\,dc_e}{c_0(1-c_0)[\exp(by_e)-1]-b(c_e-c_0)}. \tag{15}$$

Composing the Euler equations, we find that the minimum of the functional (15) will be reached when

$$\frac{c_e-c_0}{c_0(1-c_0)}=\frac{[\exp(by_e)-1]^2}{b[by_e\exp(by_e)+\exp(by_e)-1]}. \tag{16}$$

Substituting (16) into (15), we obtain the expression

$$Q=\frac{\lambda\Delta TB}{c_0(1-c_0)}\frac{\varkappa_e^*}{y_e^*}\int\limits_{c_0}^{c_{ek}}\frac{1}{b}(by_e+1-\exp(-by_e))\,dc_e,$$

which may rewritten as follows:

$$Q=\frac{\lambda\Delta TB}{c_0(1-c_0)}\frac{\varkappa_e^*}{y_e^*}\int\limits_{0}^{y_{ek}}\frac{1}{b}(by_e+1-\exp(-by_e))\frac{dc_e}{dy_e}\,dy_e. \tag{17}$$

We find the derivative dc_e/dy_e from (16)

$$\frac{dc_e}{dy_e}=c_0(1-c_0)\,by_e\frac{\exp(by_e)-\exp(-by_e)}{[by_e+1-\exp(-by_e)]^2}. \tag{17a}$$

Then instead of (17)

$$Q=\lambda\Delta TB\frac{\varkappa_e^*}{y_e^*}\int\limits_{0}^{y_{ek}}y_e\frac{\exp(by_e)-\exp(-by_e)}{by_e+1-\exp(-by_e)}\,dy_e=\lambda\Delta TB\frac{\varkappa_e^*}{y_e^*}\psi, \tag{18}$$

where the integral on the left side has been denoted by ψ.

In accordance with (5) and with allowance for the notation adopted, the ratio $\varkappa_e^*/y_e^*$ takes the form

$$\frac{\varkappa_e^*}{y_e^*}=\frac{10}{7}\frac{\sigma_e\overline{T}^2}{\alpha^2\rho D(\Delta T)^2B},$$

and instead of (18) we obtain

$$Q_{un}\equiv\frac{Q}{\sigma_e}=\frac{10}{7}\frac{\lambda\overline{T}^2\psi}{\alpha^2\rho D\Delta T}. \tag{19}$$

We will examine three cases representative of those cases of the greatest practical interest.

1. The concentration of the object is everywhere low, i.e., c << 1. In the approximation (3), this corresponds to $a = 0$, b = 1. Considering that $c_e/c_0 = q$ — the degree of separation — we obtain the following expression from (16)

$$q=\frac{y_e-1+\exp(y_e)}{y_e+1-\exp(-y_e)}, \tag{20}$$

establishing the relationship between q and the variable y_e. At the outlet of the apparatus, $y_e = y_{ek}$ and $q = q_k$.

Since q_k is prescribed by the conditions of the separation problem, then, in accordance with (20), we use it to determine y_{ek}. Values of y_{ek} are shown in Table 1. Having thus determined y_{ek}, we thereby find the upper limit of the integral in (18), which takes the form

$$\psi_1 = 2\int_0^{y_{ek}} \frac{y_e \operatorname{sh} y_e}{y_e + 1 - \exp(-y_e)}\, dy_e. \tag{21}$$

Values of this integral are shown in Table 1. When $y_{ek} > 4$, we obtain the approximation

$$\psi_1 \approx e^{y_{ek}} - 1 - 0.368\,[\mathrm{Ei}^*(y_{ek}+1) - 1.895],$$

where Ei* is a modified integral exponential function.

2. Both components are present in comparable concentrations, i.e., $0.3 < c < 0.7$. In the approximation (3), this corresponds to $b = 0$. Then instead of (16)

$$y_e = 2\frac{c_e - c_0}{c_0(1-c_0)}, \tag{22}$$

and the integral in (18) takes the form

$$\psi_0 = \int_0^{y_{ek}} y_e dy_e = \frac{1}{2}\, y_{ek}^2.$$

Substituting for y_{ek}, in accordance with (22) we obtain

$$\psi_0 = 2\left[\frac{c_{ek} - c_0}{c_0(1-c_0)}\right]^2. \tag{23}$$

3. The concentration of the object component is everywhere close to unity, i.e., $1 - c \ll 1$. In the approximation (3), this corresponds to $\alpha = 1$, $b = -1$.

Considering that the degree of separation $q = (1 - c_0)/(1 - c_e)$, we again obtain Eq. (20) from (16), and the integral in (18) takes the form

$$\psi_{-1} = 2\int_0^{y_{ek}} \frac{y_e \operatorname{sh} y_e}{\exp(y_e) + y_e - 1}\, dy_e. \tag{24}$$

The approximate expression is as follows when $y_{ek} > 4$:

$$\psi_{-1} = y_{ek}^2\left[\frac{1}{2} + \exp(-y_{ek})\left(1 + \frac{2}{y_{ek}}\right)\right] - 2.$$

Values of this integral are shown in Table 1. Now we can compare each of the above-examined variants with the ideal cascade with regard to energy efficiency.

It is known that the following is valid for an ideal cascade in which the initial concentration is maintained in the zero section:

$$Q_{\mathrm{un}}^{\mathrm{id}} = \frac{40}{7}\frac{\lambda \bar{T}^2}{\alpha^2 \rho D \Delta T} V(c_{ek}, c_0), \tag{25}$$

where the value function has the following form in the cases examined:

$$\begin{aligned} b = 1 &: V(c_{ek}, c_0) \approx q_k - 1 - \ln q_k, \\ b = 0 &: V(c_{ek}, c_0) \approx \frac{1}{2}\left[\frac{c_{ek} - c_0}{c_0(1-c_0)}\right]^2, \\ b = -1 &: V(c_{ek}, c_0) \approx \ln q_k + \frac{1}{q_k} - 1. \end{aligned} \tag{26}$$

We will define the relative efficiency as the ratio of the unit energy expenditures determined from Eqs. (25) and (19):

TABLE 1. Values of Dimensionless Height in the Outlet Section of a Thermodiffusion Unit with Transverse Flows, the Integrals of (21) and (24), and Energy Efficiency According to (27) and (46) in Relation to Degree of Separation

q_k	y_{ek}	ψ_1	ψ_{-1}	φ_1	φ_{-1}	$(y_e/\varkappa_e)_{opt}$		φ_1^*	φ_{-1}^*
						$b=1$	$b=-1$		
2	1,35	1,281	0,816	0,958	0,965	1,497	0,984	0,816	0,785
5	2,84	10,66	3,544	0,897	0,914	12,17	4,47	0,784	0,724
10	3,81	32,52	6,519	0,824	0,861	36,28	8,34	0,736	0,673
15	4,34	58,04	8,571	0,778	0,828	63,97	11,05	0,704	0,642
20	4,71	86,32	10,180	0,741	0,803	93,93	13,16	0,682	0,622
50	5,82	277,50	15,900	0,650	0,738	297,2	20,96	0,607	0,560
100	6,63	611,40	20,990	0,589	0,700	681,6	27,90	0,552	0,518
150	7,10	1039,0	23,095	0,554	0,675	1096	32,40	0,524	0,496
200	7,43	1457,0	25,480	0,532	0,655	1529	35,79	0,508	0,481
500	8,47	4209,0	34,070	0,468	0,612	4354	47,56	0,452	0,439

$$\varphi \equiv \frac{Q_{un}^{id}}{Q_{un}} = 4\,\frac{V(c_{ek},\ c_0)}{\psi}\,. \tag{27}$$

Then for each of the cases examined:

$$\varphi_1 = \frac{4}{\psi_1}(q_k - 1 - \ln q_k);\ \varphi_0 = \frac{2}{\psi_0}\left[\frac{c_{ek}-c_0}{c_0(1-c_0)}\right]^2 = 1,$$
$$\varphi_{-1} = \frac{4}{\psi_{-1}}\left(\ln q_k + \frac{1}{q_k} - 1\right). \tag{28}$$

It should be pointed out that in the second case, i.e., when the product c(1 − c) is roughly constant, the efficiency of the optimized scheme with transverse flows is the same as that of the ideal cascade.

When c << 1, then, having used the first equation of (28) and the data in Table 1, we obtain the relation shown in Fig. 2 (curve 4). If (1 − c) << 1, then the third equation of (28) and Table 1 yield curve 5 in Fig. 2. It is apparent from the latter that, up to q = 20, in the second case the energy efficiency may be 80% greater than the efficiency of the ideal cascade. Thus, in removing impurities from substances, use of a scheme with transverse flows of the product is optimal. The advantages of this scheme are particularly evident in separating mixtures in which c(1 − c) = const, where $\varphi_0 = 1$.

Let us establish the relationship between the dimensionless coordinate and the geometric characteristics of the apparatus. In the case we are examining, $\varkappa_i \to \infty$. Thus, instead of (11) and (12), we obtain

$$\frac{dc_e}{d\xi} = \frac{1}{\varkappa_e}\{c_0(1-c_0) - b(c_e-c_0)[\exp(by_e)-1]^{-1}\},$$

from which

$$d\xi = \varkappa_e\,\frac{\exp(by_e)-1}{c_0(1-c_0)[\exp(by_e)-1]-b(c_e-c_0)}\,dc_e = \frac{\varkappa_e}{c_0(1-c_0)}\,\frac{\exp(by_e)-1}{\exp(by_e)-1-b\,\dfrac{c_e-c_0}{c_0(1-c_0)}}\,\frac{dc_e}{dy_e}\,dy_e. \tag{29}$$

Allowing for (16) and (17a), we find

$$d\xi = 2\varkappa_e\,\frac{\operatorname{sh}(by_e)}{by_e+1-\exp(-by_e)}\,dy_e. \tag{30}$$

Equation (30) can be integrated in two variants: a) with the apparatus having a constant gap and variable height; b) with the apparatus having a variable gap and constant height.

In the first variant, $\varkappa_e$ = const and instead of (30)

$$\xi = 2\varkappa_e\int_0^{y_e}\frac{\operatorname{sh}(by_e)}{by_e+1-\exp(-by_e)}\,dy_e \equiv \varkappa_e I, \tag{31}$$

where I is double the value of the integral on the right side.

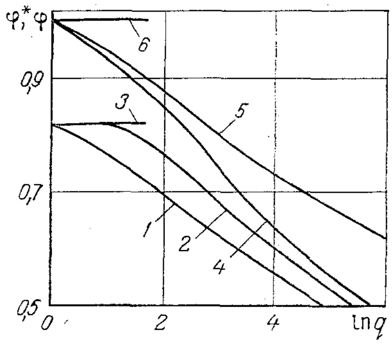

Fig. 2. Efficiency of optimized apparatus relative to ideal cascade as a function of the logarithm of the degree of separation: 1) φ_1^*; 2) φ_{-1}^*; 3) φ_0^*; 4) φ_1; 5) φ_{-1}; 6) φ_0.

The value of I with b = 1 and b = −1 is shown in Table 2. With $y_e > 4$,

$$I_1 \approx 0.368\,[\mathrm{Ei}^*(y_e+1) - 1.895].$$

When b = 0, $I = y_e$ and

$$\xi = \varkappa_e y_e \tag{32}$$

or, allowing for the notation

$$L = \frac{x}{\sigma_e}\,\frac{g^2\rho^3\beta^2\delta^7(\Delta T)^2}{9!\,\eta^2 D}, \tag{33}$$

i.e., the height of the apparatus is a linear function of the longitudinal coordinate.

If the concentration at the outlet of the apparatus c_{ek} is specified, then we can use it to determine y_{ek} in accordance with (22) and, since $\xi = 1$ in this case, allowing for (32), we have

$$\varkappa_e = \frac{1}{y_{ek}} = \frac{c_0(1-c_0)}{2(c_{ek}-c_0)}. \tag{34}$$

On the other hand,

$$\varkappa_e = \frac{\sigma_e}{H} = 6!\,\frac{\sigma_e\eta\overline{T}}{\alpha g\rho^2\beta\delta^3(\Delta T)^2 B}$$

and, using (34),

$$B = 1440\,\frac{\sigma_e\eta\overline{T}}{\alpha g\rho^2\beta\delta^3(\Delta T)^2}\,\frac{c_{ek}-c_0}{c_0(1-c_0)}. \tag{35}$$

The value of the gap should be assigned. For example, in fractionating petroleum oils, it is necessary to take $\delta \approx 0.5$–0.6 mm. When b = 1 and b = −1, we may obtain

$$B = 6!\,\frac{\sigma_e\eta\overline{T}I_k}{\alpha\rho^2 g\beta\delta^3(\Delta T)^2},$$

where I_{1k} and I_{-1k} are taken from Table 2.

In examining variant b), i.e., with L = const and $\varkappa_e$ being variable, it should be kept in mind that, from (5)

$$\varkappa_e = \frac{\varkappa_e^*}{(Ly_e^*)^{3/4}}\,y_e^{3/4},$$

and, instead of (30), (31)

$$\xi = \frac{2\varkappa_e^*}{(Ly_e^*)^{3/4}}\int\limits_0^{y_e}\frac{y_e^{3/4}\,\mathrm{sh}(by_e)\,dy_e}{by_e+1-\exp(-by_e)} = \frac{\varkappa_e^*}{(Ly_e^*)^{3/4}}\,I^*, \tag{36}$$

TABLE 2. Values of the Integrals I and I* in Relation to the Running Dimensionless Height of the Apparatus

y_e	I_1	I_{-1}	I_1^*	I_{-1}^*
0	0	0	0	0
0,2	0,2054	0,1954	0,0340	0,0317
0,4	0,4233	0,3829	0,1222	0,1098
0,6	0,6567	0,5651	0,2612	0,2174
0,8	0,9090	0,7422	0,4546	0,3527
1,0	1,1842	0,9162	0,7811	0,5134
1,2	1,4872	1,0884	1,1073	0,6982
1,4	1,8231	1,2596	1,5172	0,9066
1,6	2,1980	1,4906	1,6167	1,1383
1,8	2,6194	1,6220	2,2455	1,3935
2,0	3,0959	1,8343	3,0184	1,6724
2,2	3,6376	2,0079	3,9655	1,9753
2,4	4,2565	2,1830	5,1237	2,3025
2,6	4,9665	2,3598	6,5383	2,6542
2,8	5,7843	2,5384	8,2643	2,9906
3,0	6,7297	2,7189	10,369	3,3518
3,2	7,8257	2,9012	12,934	3,7778
3,4	9,0999	3,0852	16,059	4,2286
3,6	10,5851	3,2709	19,865	4,7040
3,8	12,3200	3,4582	24,501	5,2038
4,0	14,3508	3,6470	30,145	5,7278
4,2			37,016	6,2757

where I* is double the value of the integral on the left side, corrected for b = 1 and b = −1 in Table 2.

When b = 0, $I_0^* = (4/7)y_e^{7/4}$, and we obtain the following from (36), allowing for the notation

$$\xi = \frac{4}{7}\,\frac{\varkappa_e^*}{(Ly_e)^{3/4}}\,y_e^{7/4} = \frac{4}{7}\,9!\,\frac{\sigma_e\eta^2 DL}{\rho^3 g^2\beta^2(\Delta T)^2\delta^7 B}. \tag{37}$$

In the outlet section of the apparatus, i.e., when ξ = 1, $\delta = \delta_k$, and from (37) we obtain

$$B = \frac{4}{7}\,9!\,\frac{\sigma_e\eta^2 DL}{\rho^3 g^2\beta^2(\Delta T)^2\delta_k^7}. \tag{38}$$

Equations (37) and (38) yield

$$\frac{\delta}{\delta_k} = \xi^{-1/7}. \tag{39}$$

Since when ξ = 1 the concentration is a set quantity, according to (22)

$$y_{ek} = 2\,\frac{c_{ek}-c_0}{c_0(1-c_0)} \quad \text{or} \quad \delta_k^4 = 252\,\frac{\alpha\eta DL}{\rho g\beta\overline{T}}\,\frac{c_0(1-c_0)}{c_{ek}-c_0}. \tag{40}$$

Substitution of (40) into (38) gives

$$B = 13.01\,\frac{\sigma_e}{(\Delta T)^2}\left(\frac{\overline{T}}{\alpha}\right)^{7/4}\left(\frac{\eta}{g\beta}\right)^{1/4}\rho^{-5/4}(DL)^{-3/4}\left[\frac{c_{ek}-c_0}{c_0(1-c_0)}\right]^{7/4}. \tag{41}$$

Similarly, for the cases b = 1 and b = −1, we find

$$B = 6.769\,\frac{\sigma_e I_k^*}{(\Delta T)^2}\left(\frac{\overline{T}}{\alpha}\right)^{7/4}\left(\frac{\eta}{g\beta}\right)^{1/4}\rho^{-5/4}(DL)^{-3/4}, \tag{42}$$

where I_k^* is taken from Table 2. Knowing y_{ek}, we use the data in Table 1 to find

$$\delta_k^4 = 504\,\frac{\alpha\eta DL}{\rho g\beta\overline{T}y_{ek}}. \tag{43}$$

Thus, Eqs. (36) and (40)-(43) make it possible to calculate the geometric characteristics of the apparatus if its productivity and height and the degree of enrichment are specified.

Analysis of the above results and Eq. (36) in particular shows that optimization by variant b) is characterized by only a slight change in the gap size over a long section of the apparatus length (ξ > 0.1). Considering the difficulties of making a unit out of elements with gap sizes differing altogether by hundredths of a millimeter, such a variant could hardly be recommended. Variant a), depicted in Fig. 1c, is more expedient. Here, the cascade consists of identical elements of small height with even gaps.

In conclusion, we will present yet one more method of optimization, based on examination of the formula

$$Q^*_{un} = \frac{10}{7} \frac{\lambda \overline{T}^2}{\alpha^2 \rho D \Delta T} \frac{y_e}{\varkappa_e}, \tag{44}$$

obtained from (3a) after transformations. It is assumed in this formula that L = const and δ = const over the entire length of the unit.

As before, limiting ourselves to the case $\varkappa_i \to \infty$, we obtain the following expressions for $\varkappa_e$ from (1) or (2):

$$\begin{gathered} b=1: \varkappa_e = -\left\{[\exp(y_e)-1]\ln\left[1-\frac{q-1}{\exp(y_e)-1}\right]\right\}^{-1}, \\ b=0: \varkappa_e = -\left\{y_e \ln\left[1-\frac{c_{ek}-c_0}{y_e c_0(1-c_0)}\right]\right\}^{-1}, \\ b=-1: \varkappa_e = -\left\{[1-\exp(-y_e)]\ln\left[1-\frac{q-1}{q(1-\exp(-y_e))}\right]\right\}^{-1}, \end{gathered} \tag{45}$$

Substituting these expressions into (44) and equating the derivative dQ^*_{un}/dy_e to zero, we find the values of $y_{e,opt}$ at which Q^*_{un} will have a minimum value. Then having determined $(y_e/\varkappa_e)_{opt}$, we can calculate the efficiency of the thus-optimized apparatus compared to the ideal cascade. Using (26), by analogy with (27) we obtain

$$\varphi^* \equiv Q^{id}_{un}/Q^*_{un} = 4V(c_e,\ c_0) \Big/ \left(\frac{y_e}{\varkappa_e}\right)_{opt}. \tag{46}$$

Values of φ^*_1 and φ^*_{-1} are shown in Table 1. For the case b = 0, throughout the entire range of $c_{ek} - c_0$ permissible for the given approximation φ^*_0 remains roughly constant, equal to 0.814.

It is apparent from Fig. 2, which also shows the dependence of φ* on log q, that φ > φ* and, thus, the method of optimization based on (44) has no disadvantages except for the case c << 1 with q > 50, when φ is not much larger than φ* (curves 2 and 4). This is because the reduction in energy efficiency in this case is compensated for by the simpler design of the apparatus.

NOTATION

c, mass concentration; ξ = x/B; x, longitudinal coordinate; B, total length of apparatus; $y_e = 504\alpha\eta DL/\rho g\beta\delta^4\overline{T}$; α, thermal diffusion constant; η, D, β, dynamic viscosity, diffusion coefficient, and coefficient of cubical expansion; L, height of apparatus; ρ, density; δ, depth of gap; $\overline{T}$, mean temperature in gap; ϰ = σ/H; σ, productivity of apparatus (extraction); $H = \alpha\rho^2 g\beta\delta^3(\Delta T)^2 B/\eta\overline{T}$; ΔT, difference in temperatures between hot and cold surfaces; Q, heat flow rate; λ, thermal conductivity; q, degree of separation; Indices: e, i, enriched and depleted products; k, value at apparatus outlet.

LITERATURE CITED

1. A. V. Suvorov and G. D. Rabinovich, "Theory of thermodiffusion apparatus with transverse flows," Inzh.-Fiz. Zh., 41, No. 2, 231-238 (1981).
2. A. V. Suvorov, G. D. Rabinovich, M. A. Bukhtilova, et al., "Principles of design of units for separating petroleum products by thermal diffusion," in: Heat and Mass Transfer and the Transport Properties of Substances [in Russian], ITMO AN BSSR (A. V. Lykov Institute of Heat and Mass Transfer of the Belorussian Academy of Sciences), Minsk (1978).